

**Measurement of Capital Stocks, Consumption of Fixed Capital, and Capital Services:  
Report on a Presentation to the Central American *Ad Hoc* Group on National Accounts  
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**I. Overview of Capital Measurement Issues**

Estimates of fixed capital stocks are needed for preparation of national balance sheets, for making the estimates of consumption of fixed capital (CFC) that are part of the measurement net national income, and for making the estimates of capital services required for measurement of multi-factor productivity change.

We consider first the question of how to estimate net changes in stocks of various types of fixed capital assets, deferring discussion of the question of how to estimate their initial levels. Two approaches are possible, one based on time series on gross investment by class of asset, and another based on data on stocks of assets from censuses or surveys, possibly from administrative sources. (Some examples of administrative sources are records of motor vehicle registrations or property tax records on structures. If a survey is used, one of the pitfalls is that businesses tend to report historic cost of capital assets, which will need to be adjusted for price change to obtain meaningful constant-dollar measures.) Among the possible ways to implement the gross investment approach are the geometric permanent inventory model (PIM), straight-line depreciation, and hyperbolic depreciation. For methods other than the geometric PIM, asset retirement profiles, such as those based on the studies of Winfrey, must be used. For the PIM estimates of depreciation rates based on a study by Hulten and Wykoff are available. These estimates can be used to calculate consumption of fixed capital (CFC, or depreciation), which must be deducted from gross investment to calculate net investment.

Before calculating CFC and net investment, “historic cost” measures of investment must be converted into constant price measures. After doing all the calculations in constant prices, the results can be reflated to obtain current-price measures. One use of current-price CFC is the calculation of net income measures for businesses that own capital assets. Effects of quality change in capital assets (such as computers) are adjusted for in the price indexes. In calculations of CFC using a geometric PIM, BEA uses a “mid-year convention” for new investment, which means that gross investment during a year is subject one-half year of depreciation.

Initial levels of stocks of various classes of capital assets can be estimated from long time series on gross investment or from censuses or surveys of these stocks at some point in time. Estimation of initial stocks is a challenge for Central American countries because they do not

have long time series on gross investment or censuses of stocks of fixed assets. By assuming an average growth rate of investment in the years preceding the one when the data on a class of asset first become available, a simple model based on the geometric PIM can be used to estimate the level of the initial stock. In the appendix, we show that the level of the stock  $V_0$  at the end of the initial year can be estimated as:

$$V_0 = I_0 \frac{g + 1}{g + d} \quad (1)$$

where  $I_0$  denotes the observed level of investment in the initial year of data collection,  $d$  denotes the constant geometric rate of depreciation, and  $g$  denotes the average growth rate of investment in the years preceding year 0. If the current year  $t$  is far enough away from year 0 and if  $d$  is not small, the inaccuracy in the estimate of the current year stocks caused by the lack of data for years before 0 will be small. Often, this will be true for equipment and software, but not for structures, which have small values of  $d$ . Thus, direct measures of the stock of structures are desirable unless the time series on gross investment in various types of structure extends far back in time.

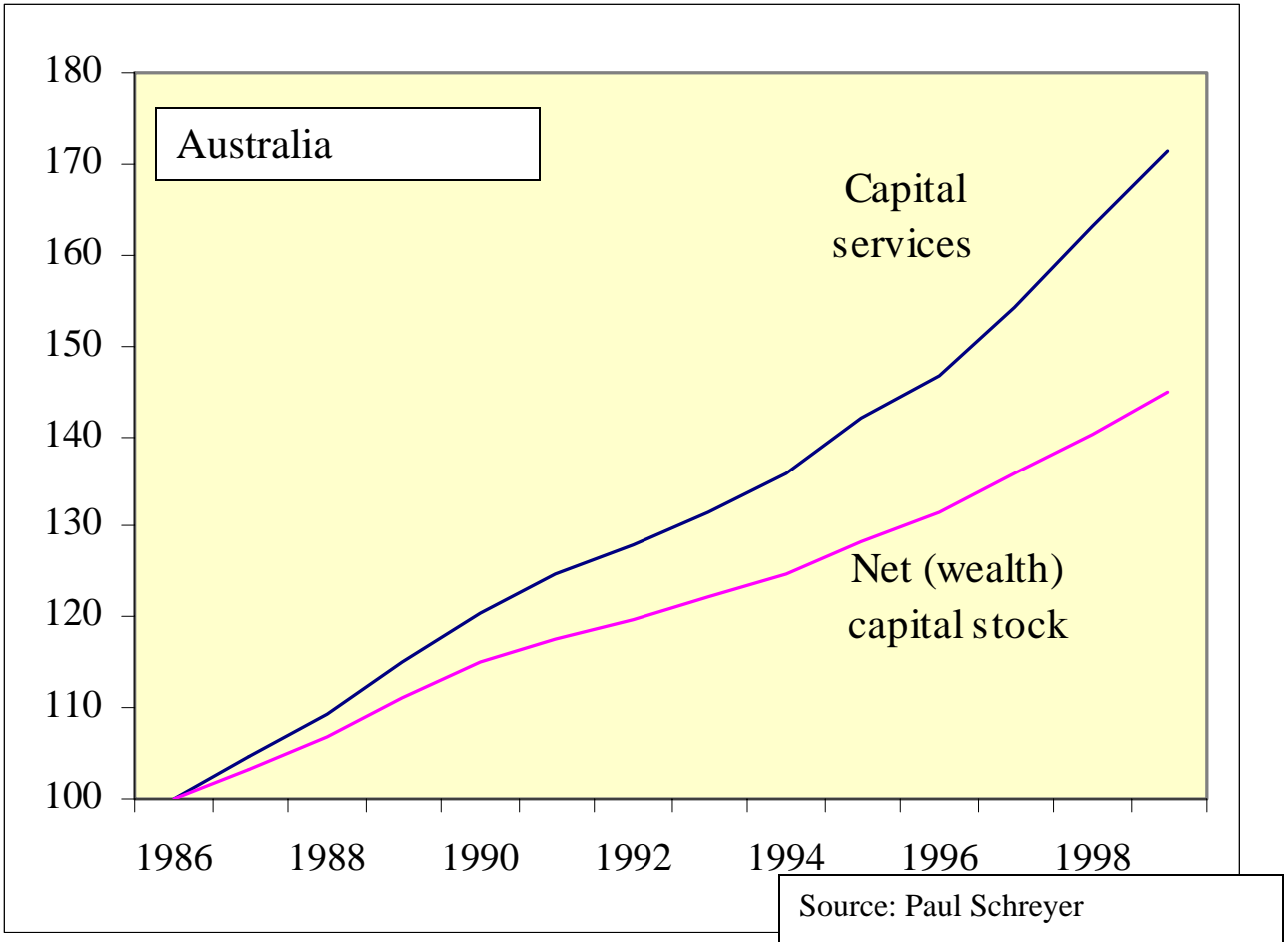
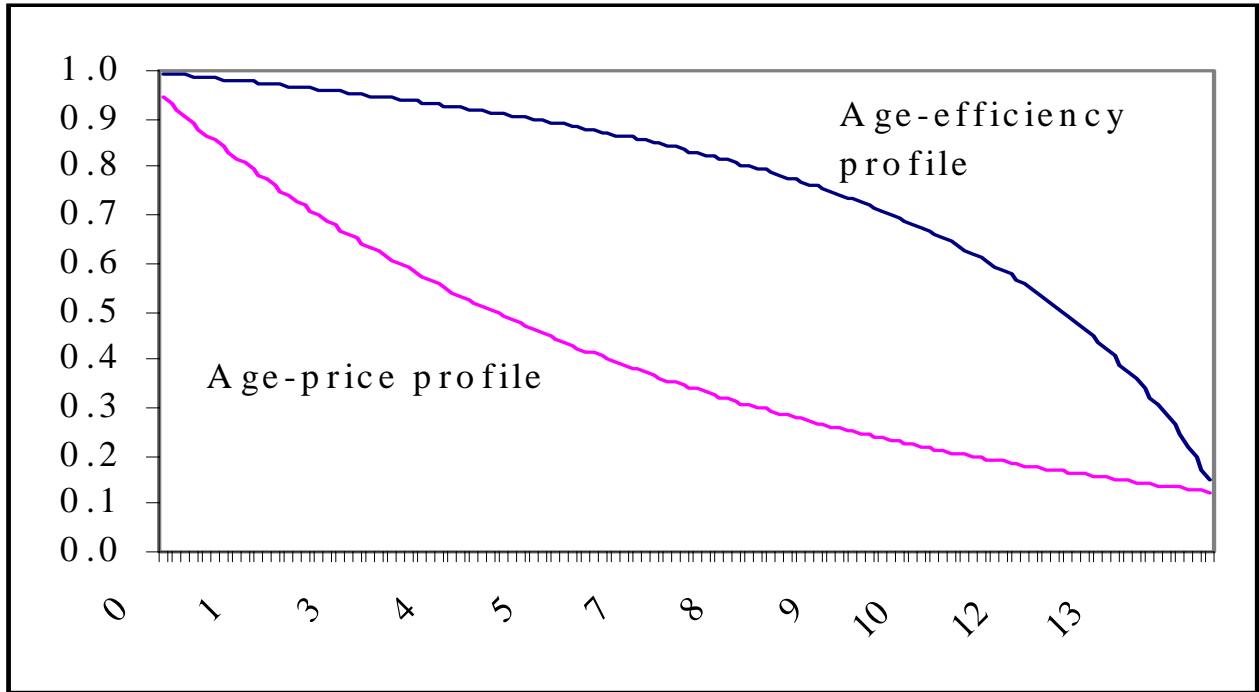
Capital services, which are useful for evaluating multi-factor productivity growth, can be calculated by means of a user-cost of capital formula. If  $r$  denotes a rate of interest and  $\Delta p_V$  denotes an expected holding gain on the asset caused by changes in the price of that asset then the user cost  $uc_t$  equals:

$$uc_t = V_t(r + d - \Delta p_V). \quad (2)$$

In practice, in applying the user cost formula, it is convenient to combine  $r$  and  $\Delta p_V$  to obtain a real rate of interest. With a constant real rate of interest, and a geometric PIM with an infinite service life and constant  $d$ , the growth rate of capital services will equal the growth rate of the capital stock. However, analysis of productivity is often done using a hyperbolic depreciation model or a one-hoss shay model. With these models, we can distinguish the “efficiency” or productive capital stock from the wealth capital stock. For example, the efficiency of a one-hoss shay type asset does not decline until it is retired from service, but the price of the asset declines smoothly over its service life to reflect its remaining years of service. The chart on the following page shows the relationship between the age-price profile and the age-efficiency profile  $V_t^e$  for the hyperbolic model with  $\beta = 0.5$ . According to this model:

$$V_t^e = V_0^e(T-t)/(T-\beta t) \quad (3)$$

In recent years in many countries, rapid growth in investment in equipment, such as computers, and in software has shortened the average service life and raised the average depreciation rate of the capital stock. Capital services, and the efficiency capital stock, have therefore risen faster than the wealth capital stock. An illustration of this for the case of Australia (reproduced from a presentation by Paul Schreyer) is also found on the next page.



A presentation by Diewert, Harrison and Schreyer provided an illustration of a possible scheme for estimation of the capital stock, CFC, and capital services in national accounts. It is reproduced at the end of this document.

## **II. Introduction of Capital Measures in Central American National Accounts**

The Central American countries do not have long time series on gross investment, and even if such series were available, conversion from historic-cost price to constant-year prices would often be problematical. Recently, the Secretary of the Central American Monetary Council sent a questionnaire to the heads of Central American national accounts departments aiming to determine, among other aspects, the detail and length of series on gross investment. Up to date only two countries of the six countries have responded. Guatemala has stated that even though they have data from 1958, this information is not detailed enough and they have some concerns about quality. Costa Rica has measures of stocks of structures and machinery and equipment for businesses and the public sector since 1966 at current and constant prices, but detail is a concern, however, at least for machinery and equipment and some kind of structures, more detailed data can be reconstructed in order to achieve acceptable figures of stocks by categories; price figures must be improved. However it is important to note that the feasibility of making capital stock estimates in Latin America has been demonstrated by the success of the Instituto Nacional de Estadística y Censos of Argentina—see *Estimación del Stock de Capital Fijo del República Argentina, 1990-2003: Fuentes, Métodos y Resultados*, Estudios 39, INDEC (Buenos Aires), 2004.

Some Central American countries plan to begin by estimating stocks of fixed assets by somewhat detailed categories; no effort to estimate capital services or multi-factor productivity based on estimates of capital services is planned at this time. The capital stock estimates will be used to develop estimates of consumption of fixed capital needed for measurement of the net operating surplus of business and if feasible also for non-market producers. The estimates of capital stocks and CFC based on data on gross investment will be developed using the geometric permanent inventory model. The initial level of the stocks will be estimated for most items using the approach of equation (1) above, but for a few types of items administrative records or surveys will be used.

In Central American countries, capital equipment and software items are predominately imported, so data sets on imports that contain time series on equipment and software that can be used to develop estimates of that cover a large portion of equipment stocks. These estimates of imported equipment stocks will be augmented by estimates of domestically produced equipment; own-account production of software will be omitted. Since equipment and software items have high depreciation rates, reasonably accurate estimates can be calculated using equation (1) above even without a long history of investment.

Partial data are also available on government expenditures on equipment and software and on many categories of structures. In some cases, expenditure data from budget information can be used. These efforts will be made in the medium term, when most of the countries will have ended the development of figures for supply and use tables, in which they are now engaged.

## Appendix: Derivation of Equation (1) as Method of Estimating the Initial Stock of a Capital Asset Item

**PROPOSITION:** For some particular type of fixed capital asset, let  $l_0$  denote the observed level of investment in the initial year of data collection, let  $d$  denote the constant geometric rate of depreciation, and let  $g$  denote that average growth rate of investment in the years preceding year 0. Then the level of the capital stock for this type of asset at the end of year 0, denoted by  $V_0$ , may be estimated as:

$$V_0 = l_0 \frac{g + 1}{g + d}.$$

**PROOF:** If constant-price gross investment in year 0 is  $l_0$ , then constant-price investment in year  $-1$  is  $l_0/(1+g)$ . Assuming that depreciation starts in the year following the investment (so that the mid-year convention does not apply), the amount of this investment still in the capital stock at the end of year 0 is  $l_0(1-d)/(1+g)$ . Similarly, the amount of investment that occurred in year  $-2$  that is still in the capital stock at the end of year 0, is  $l_0[(1-d)/(1+g)]^2$ . Combing terms for investment in year 0 and in all earlier years, we have:

$$V_0 = l_0 \left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\}.$$

Note that:

$$\left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\} \frac{1+g}{1-d} = \frac{1+g}{1-d} + \left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\}.$$

Moving the second term on the right hand side to the left hand side, we have:

$$\left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\} \left\{ \frac{1+g}{1-d} - 1 \right\} = \frac{1+g}{1-d}$$

$$\left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\} \left\{ \frac{g+d}{1-d} \right\} = \frac{1+g}{1-d}$$

$$\left\{ 1 + \frac{1-d}{1+g} + \left[ \frac{1-d}{1+g} \right]^2 + \dots \right\} = \frac{1+g}{g+d}.$$

